

# SPIN DEPENDENT MOMENTUM DISTRIBUTIONS IN DEFORMED NUCLEI

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## Abstract

We study the properties of the spin dependent one body density in momentum space for odd- $A$  polarized deformed nuclei within the mean field approximation. We derive analytic expressions connecting intrinsic and laboratory momentum distributions. The latter are related to observable transition densities in  $\mathbf{p}$ -space that can be probed in one nucleon knock-out reactions from polarized targets. It is shown that most of the information contained in the intrinsic spin dependent momentum distribution is lost when the nucleus is not polarized. Results are presented and discussed for two prolate nuclei,  $^{21}\text{Ne}$  and  $^{25}\text{Mg}$ , and for one oblate nucleus,  $^{37}\text{Ar}$ . The effects of deformation are highlighted by comparison to the case of odd- $A$  nuclei in the spherical model.

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# I. Introduction

Momentum distributions in nuclei are a subject of increasing interest at both nucleon and parton levels. Reviews of theory and models on nucleon momentum distributions can be found in Refs. [1, 2]. Experimental information on nucleon momentum distributions can be gained from one nucleon knock-out in nuclear photo-absorption, as well as by  $A(x, x'N)B$  reactions<sup>[3]</sup> –where a hadronic or leptonic beam,  $x$ , scatters and knocks out one nucleon from a nuclear target,  $A$ . As pointed out in Ref. [1] the interpretation of information on momentum distributions is not free from ambiguities because, at present, neither experimental data nor theoretical considerations are sufficiently comprehensive. Yet fairly reliable information is already available on nucleon momentum distributions from electron scattering in the quasi-elastic region<sup>[4]</sup>, as well as on parton distributions from lepton scattering in the deep inelastic region<sup>[5]</sup>. Most of the work on complex nuclei has been devoted to spherical nuclei, but in the last years several questions have emerged concerning the role of nuclear deformation in quasi-elastic electron scattering and in deep inelastic scattering. In the latter, it has been suggested to use polarized deformed nuclei to study the degree of transparency in different directions<sup>[6]</sup>; in the first it has been questioned whether coincidence measurements may be sensitive to nuclear deformation.

Investigations of quasi-elastic electron scattering from even-even deformed nuclei have been carried out both theoretically and experimentally. Starting from mean field calculations with increasing degree of sophistication, we studied the properties of momentum distributions in the intrinsic ground state of several even-even axially symmetric deformed nuclei. We investigated not only effects of deformation but also effects of pairing and of short range correlations<sup>[7, 8, 9]</sup>. In a search for deformation effects, coincidence  $(e, e'p)$  experiments on spherical and deformed Nd isotopes were performed at NIKHEF<sup>[10]</sup>. More recently we have also initiated studies of deformation effects in quasi-elastic electron scattering from polarized deformed nuclei<sup>[11]</sup>. With progress made in developing polarized beams and targets, as well as in polarimeters designs (in addition to that on CW linac facilities), the study of spin degrees of freedom is becoming a major theme in Nuclear Physics either at low, intermediate or high energies. In this paper we attempt to lay a basis for such studies in odd- $A$  deformed nuclei.

In general the one-body density (either in  $\mathbf{r}$ -space or in  $\mathbf{p}$ -space) is a two by two matrix in spin space. For closed shell (spin saturated) nuclei the one-body density is proportional to the unit matrix in spin space, i.e., the momentum distribution is independent of spin. The same is true for even-even axially symmetric deformed nuclei in the mean field approximation due to time reversal invariance of the intrinsic ground state. However, for an odd- $A$  deformed nucleus the intrinsic ground state is no longer time reversal invariant and the one-body density matrix is in general non diagonal in spin space. Hence **scalar** and **vector** momentum distributions can be defined in the intrinsic frame, that contain all the information on the intrinsic spin dependent momentum distribution. This paper is devoted to the study of these new momentum distributions.

It is important to realize that in order to access to this new information one needs to relate the observable spin dependent momentum distributions of the polarized nucleus in the laboratory frame with the intrinsic scalar and vector momentum distributions. A central issue of this paper is to establish such relations.

We study properties of scalar and vector momentum distributions within the self-consistent mean field approximations assuming axially symmetric and time-reversal invariant mean fields. To this end we present and discuss results of density dependent Hartree-Fock (DDHF) calculations for two nuclei  $^{21}\text{Ne}$  and  $^{37}\text{Ar}$  that have ground states with equal spin and parity ( $3/2^+$ ) but have quite different intrinsic structure. Results for  $^{25}\text{Mg}$  are also discussed. The self-consistent mean fields for  $^{21}\text{Ne}$  and  $^{25}\text{Mg}$  are prolate whereas that for  $^{37}\text{Ar}$  is oblate, and the  $K$ -values are different for each nucleus. The role of deformation is further highlighted by comparison to the limit of a spherical mean field, i.e., to the case of odd-A nuclei with a single-nucleon outside closed shells.

The paper is organized as follows: in Section II the intrinsic scalar and vector momentum distributions are defined and their properties are studied. In Section III we define scalar and vector momentum distributions for the polarized nucleus in the laboratory frame, we establish their connection with the intrinsic ones, as well as with observable transition densities in  $\mathbf{p}$ -space. In this section the spherical limit is also discussed and comparisons with results on the deformed nuclei are presented. Section IV summarizes our conclusions.

## II. Intrinsic Momentum Distributions

To study spin dependent momentum distributions in axially symmetric deformed nuclei our starting point are the single-particle Hartree-Fock (HF) solutions in coordinate space, which are characterized by the angular momentum projection along the symmetry axis ( $\Omega_i$ ) and by the parity ( $\pi_i$ ). The HF wave functions are expressed in the intrinsic system with the  $z$  axis the symmetry axis and the  $(x, y)$  plane the symmetry plane. The states  $i, \bar{i}$  degenerate in energy are written as<sup>[12]</sup>

$$\Phi_i(\mathbf{r}, s, q) = \chi_{q_i}(q) \left[ \phi_i^{(+)}(r_\perp, z) e^{i\varphi\Lambda^-} \chi_+(s) + \phi_i^{(-)}(r_\perp, z) e^{i\varphi\Lambda^+} \chi_-(s) \right], \quad (1)$$

$$\Phi_{\bar{i}}(\mathbf{r}, s, q) = \chi_{q_i}(q) \left[ -\phi_i^{(-)}(r_\perp, z) e^{-i\varphi\Lambda^+} \chi_+(s) + \phi_i^{(+)}(r_\perp, z) e^{-i\varphi\Lambda^-} \chi_-(s) \right] \quad (2)$$

where  $\chi_{q_i}(q)$ ,  $\chi_\pm(s)$  are isospin and spin functions,  $\bar{i}$  is the time reverse of  $i$ ,  $\Lambda^\pm = \Omega_i \pm 1/2 \geq 0$ , and  $r_\perp, z, \varphi$  are the cylindrical coordinates of  $\mathbf{r}$ . The wave functions  $\phi_i^{(\pm)}$  are expanded into eigenfunctions of an axially deformed Harmonic Oscillator, characterized by the set of quantum numbers  $\{\alpha\} = \{n_\perp, n_z, \Lambda, \Sigma\}$ , with expansion coefficients  $C_\alpha^i$ . Taking the Fourier transforms of Eqs. (1) and (2) we get the single-particle HF wave functions in momentum space

$$\tilde{\Phi}_i(\mathbf{p}, s) = \tilde{\phi}_i^{(+)}(p_\perp, p_z) e^{i\varphi_p\Lambda^-} \chi_+(s) + \tilde{\phi}_i^{(-)}(p_\perp, p_z) e^{i\varphi_p\Lambda^+} \chi_-(s), \quad (3)$$

$$\tilde{\Phi}_{\bar{i}}(\mathbf{p}, s) = \left[ -\tilde{\phi}_i^{(-)}(p_\perp, p_z) e^{-i\varphi_p\Lambda^+} \chi_+(s) + \tilde{\phi}_i^{(+)}(p_\perp, p_z) e^{-i\varphi_p\Lambda^-} \chi_-(s) \right] \pi_i \quad (4)$$

with  $p_\perp, p_z, \varphi_p$  the cylindrical coordinates of  $\mathbf{p}$ . Here and in what follows we omit isospin to abbreviate the notation. The wave functions  $\tilde{\phi}_i^{(\pm)}$  are given by

$$\tilde{\phi}_i^{(\mp)}(p_\perp, p_z) = \sum_{n_\perp, n_z} C_{\alpha^\pm}^i \tilde{\phi}_{\alpha^\pm}(p_\perp, p_z) \quad (5)$$

where  $\alpha^\pm$  stands for  $\{n_\perp, n_z, \Lambda = \Lambda^\pm\}$  and the sum extends over basis states with major quantum numbers ( $N = 2n_\perp + n_z + \Lambda$ ) even or odd for  $\pi_i = +1$  and  $\pi_i = -1$ , respectively. In the cylindrical basis

$$\tilde{\phi}_{\alpha^\pm}(p_\perp, p_z) = \frac{(-i)^N \sqrt{\pi_i}}{\sqrt{2\pi}} \psi_{n_\perp}^{\Lambda^\pm}(p_\perp) \psi_{n_z}(p_z) \quad (6)$$

with

$$\psi_{n_\perp}^\Lambda(p_\perp) = \left[ \frac{2n_\perp!}{\beta_\perp^2 (n_\perp + \Lambda)!} \right]^{1/2} \eta^{\Lambda/2} e^{-\eta/2} L_{n_\perp}^\Lambda(\eta), \quad (7)$$

$$\psi_{n_z}(p_z) = \left[ \frac{1}{\beta_z \sqrt{\pi} 2^{n_z} n_z!} \right]^{1/2} e^{-\xi^2/2} H_{n_z}(\xi) \quad (8)$$

with  $L_{n_\perp}^\Lambda$  and  $H_{n_z}$  associated Laguerre and Hermite polynomials respectively,  $\eta = p_\perp^2/\beta_\perp^2$ ,  $\xi = p_z/\beta_z$  and  $\beta_\perp, \beta_z$  the inverse of the harmonic oscillator length parameters ( $\beta_\perp = (m\omega_\perp/\hbar)^{1/2}$ ,  $\beta_z = (m\omega_z/\hbar)^{1/2}$ ). For later use it is convenient to expand the momentum dependent single-particle wave functions in a spherical basis. We write

$$\tilde{\Phi}_i(\mathbf{p}, s) = \sum_{\ell j} \mathcal{Y}_{\ell j}^{\Omega_i}(\hat{p}, s) \tilde{\phi}_i^{\ell j}(p) \quad (9)$$

$$\tilde{\Phi}_{\bar{i}}(\mathbf{p}, s) = \sum_{\ell j} (-1)^{\Omega_i - j} \mathcal{Y}_{\ell j}^{-\Omega_i}(\hat{p}, s) \tilde{\phi}_i^{\ell j}(p) \quad (10)$$

with

$$\mathcal{Y}_{\ell j}^m(\hat{p}, s) = \sum_{\Lambda \Sigma} \langle \ell \Lambda \frac{1}{2} \Sigma | j m \rangle Y_\ell^\Lambda(\hat{p}) \chi_\Sigma(s) \quad (11)$$

and

$$\tilde{\phi}_i^{\ell j}(p) = \sum_\alpha C_\alpha^i \langle \ell \Lambda \frac{1}{2} \Sigma | j \Omega_i \rangle R_\ell^\alpha(p) \quad (12)$$

$$\begin{aligned} R_\ell^\alpha(p) &= \int d\Omega_p Y_\ell^{\Lambda*}(\Omega_p) \tilde{\phi}_\alpha(p_\perp, p_z) e^{i\varphi_p \Lambda} \\ &= (-i)^{N+2\Lambda} \sqrt{\pi_i} \left[ \frac{(2\ell+1)(\ell-\Lambda)!}{2(\ell+\Lambda)!} \right]^{1/2} \int_0^\pi \sin \theta_p d\theta_p P_\ell^\Lambda(\cos \theta_p) \psi_{n_\perp}^\Lambda(p \sin \theta_p) \psi_{n_z}(p \cos \theta_p). \end{aligned} \quad (13)$$

We represent by  $\Psi_k$  the intrinsic ground state of an odd-A nucleus with odd nucleon in state  $k$ .  $\tilde{\Phi}_k$  represents the wave function in momentum space for the state  $k$ , and  $\pi_k, \Omega_k = K$  are its quantum numbers. The intrinsic one-body spin dependent momentum distribution is then given by

$$\begin{aligned}
M_{\Sigma\Sigma'}(\mathbf{p}) &= \langle \Psi_k | a_{\mathbf{p}\Sigma}^+ a_{\mathbf{p}\Sigma'} | \Psi_k \rangle \\
&= \sum_{i \neq k} v_i^2 \left[ \langle \chi_{\Sigma'} | \tilde{\Phi}_i(\mathbf{p}, s) \rangle \langle \tilde{\Phi}_i(\mathbf{p}, s) | \chi_{\Sigma} \rangle + \langle \chi_{\Sigma'} | \tilde{\Phi}_{\bar{i}}(\mathbf{p}, s) \rangle \langle \tilde{\Phi}_{\bar{i}}(\mathbf{p}, s) | \chi_{\Sigma} \rangle \right] \\
&+ \langle \chi_{\Sigma'} | \tilde{\Phi}_k(\mathbf{p}, s) \rangle \langle \tilde{\Phi}_k(\mathbf{p}, s) | \chi_{\Sigma} \rangle
\end{aligned} \tag{14}$$

with  $v_i^2$  occupation probabilities. The last term in this equation gives the contribution from the odd nucleon, while the first term gives the contribution from the nucleons in the even–even core. Decomposing this matrix into a **scalar** ( $\overline{M}$ ) and a **vector** ( $\widehat{M}$ ) in spin space

$$\overline{M} = \text{Tr} [M(\mathbf{p})] \tag{15}$$

$$\widehat{M} = \text{Tr} [M(\mathbf{p})\boldsymbol{\sigma}] \tag{16}$$

it is easy to show that the even–even core contributes only to the scalar momentum distribution,

$$\begin{aligned}
\overline{M}(\mathbf{p}) &= \sum_{i \neq k} 2v_i^2 \left( |\tilde{\phi}_i^{(+)}(p_{\perp}, p_z)|^2 + |\tilde{\phi}_i^{(-)}(p_{\perp}, p_z)|^2 \right) \\
&+ |\tilde{\phi}_k^{(+)}(p_{\perp}, p_z)|^2 + |\tilde{\phi}_k^{(-)}(p_{\perp}, p_z)|^2,
\end{aligned} \tag{17}$$

while the vector momentum distribution is made up entirely by the odd nucleon,

$$\widehat{M}_z(\mathbf{p}) = |\tilde{\phi}_k^{(+)}(p_{\perp}, p_z)|^2 - |\tilde{\phi}_k^{(-)}(p_{\perp}, p_z)|^2 \tag{18}$$

$$\widehat{M}_x(\mathbf{p}) \equiv \cos \varphi_p \widehat{M}_{\perp}(p_{\perp}, p_z) \tag{19}$$

$$\widehat{M}_y(\mathbf{p}) \equiv \sin \varphi_p \widehat{M}_{\perp}(p_{\perp}, p_z) \tag{20}$$

with

$$\widehat{M}_{\perp}(p_{\perp}, p_z) = 2\tilde{\phi}_k^{(+)}(p_{\perp}, p_z)\tilde{\phi}_k^{(-)}(p_{\perp}, p_z) . \tag{21}$$

In previous work<sup>[8, 9]</sup> we studied momentum distributions in even–even deformed nuclei. For spin–saturated even–even nuclei the vector momentum distribution is zero in the intrinsic ground state and only the scalar momentum distribution remains. To connect with the notation in Refs. [8, 9] we denote by  $n(\mathbf{p})$  the contribution from the even–even core to  $\overline{M}(\mathbf{p})$ , and by  $n_{\lambda}(p)$  its multipoles. The contribution from the odd–nucleon is denoted by  $n^{\text{odd}}(\mathbf{p})$

$$\overline{M}(\mathbf{p}) = n(\mathbf{p}) + n^{\text{odd}}(\mathbf{p}) = \sum_{\lambda=\text{even}} P_{\lambda}(\cos \theta_p) \left[ n_{\lambda}(p) + n_{\lambda}^{\text{odd}}(p) \right] \equiv \sum_{\lambda=\text{even}} P_{\lambda}(\cos \theta_p) \overline{M}_{\lambda}(p) . \tag{22}$$

A deformation parameter in momentum space  $\beta^p$  was introduced in Refs. [8, 9] defined in analogy to the standard<sup>[13]</sup> definition of the mass quadrupole deformation parameter  $\beta^r$ ,

$$\beta^p = \sqrt{\frac{4\pi}{5}} \frac{\int n(\mathbf{p}) p^2 P_2(\cos \theta_p) d\mathbf{p}}{\int n(\mathbf{p}) p^2 d\mathbf{p}} \quad (23)$$

For odd-A nuclei the same definition applies replacing  $n(\mathbf{p})$  by  $\overline{M}(\mathbf{p})$ .

In what follows we show scalar and vector intrinsic momentum distributions of  $^{21}\text{Ne}$  and  $^{37}\text{Ar}$ . The results presented have been obtained using the SKA effective interaction<sup>[14]</sup> and the McMaster version of the deformed Hartree-Fock code<sup>[15]</sup> that follows closely the procedure of Ref. [12]. The HF results for binding energies as well as for the charge and mass quadrupole moments and r.m.s. radii are summarized in table I. Also given in the table are the results for the even-even isotopes  $^{20}\text{Ne}$  and  $^{36}\text{Ar}$ . Important differences between the structures of Ne and Ar isotopes are that the former is prolate while the latter is oblate and that the unpaired neutrons in  $^{21}\text{Ne}$  and in  $^{37}\text{Ar}$  sit in orbitals with  $\Omega_k = 3/2$  and with  $\Omega_k = 1/2$ , respectively, while both nuclei have  $J = 3/2$  in their ground states. Also given in table I are the moments of inertia and the decoupling parameter (defined as in Ref. [16]). It can be checked that with these parameters the angular momentum sequences of the ground state bands in  $^{21}\text{Ne}$  and in  $^{37}\text{Ar}$  are well described<sup>[17]</sup> when compared to experiment<sup>[18]</sup>. The moments of inertia have been calculated using the cranking formula.

In Figure 1 we show in three-dimensional plots the contributions to the scalar momentum distribution in  $^{21}\text{Ne}$  (top) and  $^{37}\text{Ar}$  (bottom) from the core (left) and from the odd nucleon (right). As already mentioned, the odd nucleons in  $^{21}\text{Ne}$  and  $^{37}\text{Ar}$  have  $\Omega_k = 3/2$  and  $1/2$ , respectively. Clearly the odd nucleon contributions are quite different in  $^{21}\text{Ne}$  and in  $^{37}\text{Ar}$ , this is due to the different  $K$  values and mean field deformations. We shall come back to this point later on. It is also interesting to remark the difference between odd nucleon and core nucleons contributions. An important difference between the momentum distribution of the core ( $n(\mathbf{p})$ ) and the momentum distribution of the odd nucleon ( $n^{\text{odd}}(\mathbf{p})$ ) is that in the first case the monopole term,  $n_0(p)$ , is dominant while in the second the  $\lambda = 2$  and higher multipoles are comparable to the monopole term. In Figure 2 we show the  $\lambda = 0, 2$  multipoles of the even-even core of Ne and Ar. It is worth pointing out that the isotropy condition found in previous work on even-even deformed nuclei<sup>[8, 9]</sup> is also met here for the even-even cores, in the sense that when we compute the deformation parameter  $\beta^p$  we get  $\beta^p \approx 0$  (see table II), whereas the corresponding parameter in  $r$ -space is large, consistently with the experimental deformation. Experimentally<sup>[13, 19]</sup> the quadrupole moments are  $51.4 \pm 3.8$ ,  $58.5 \pm 2.6$ , and  $-38.5 \pm 21$  fm<sup>2</sup>, which correspond to  $\beta^r$  values of  $0.45 \pm 0.03$ ,  $0.51 \pm 0.02$ , and  $-0.15 \pm 0.08$ , for  $^{21}\text{Ne}$ ,  $^{20}\text{Ne}$ , and  $^{36}\text{Ar}$ , respectively.

Actually, as seen in table II,  $\beta^p$  is close to zero not only for the even-even, but also for the odd-A nuclei. This important property follows from the major shell admixtures in the single-particle wave functions that result from the self-consistent mean field<sup>[8, 9]</sup>. Interferences between different major shells have opposite sign in  $r$ -space and in  $p$ -space, and make it possible to reach configurations with large equilibrium deformations and

minimum kinetic energies. To illustrate the role of N–admixture in momentum space we show on the right hand side of Fig. 2 the results obtained with the Nilsson model when the Nilsson hamiltonian<sup>[20]</sup> is diagonalized in the entire basis space ( $N \leq 7$ ) and when it is diagonalized within each major shell subspace (the  $\chi$ ,  $\mu$  parameters have been chosen as in Ref. [21]). Clearly, the results for  $n_2(p)$  are substantially different in both cases. When  $\Delta N$  admixtures are taken into account the results are similar to the ones obtained in DDHF: the oscillations in  $n_2(p)$  are such as to minimize the  $\beta^p$  value. When  $\Delta N$  admixtures are neglected the  $\lambda = 2$  multipole of the density in momentum space has the same shape as the  $\lambda = 2$  multipole of the density in  $r$ –space, and the  $\beta^p$  value turns out to be the same as the deformation parameter in coordinate space  $\beta^r$  (see table II). A similar situation is found for the  $\lambda = 2$  multipoles of the odd nucleon momentum distributions  $n^{\text{odd}}(\mathbf{p})$ .

The vector momentum distribution components  $\widehat{M}_z(\mathbf{p})$  and  $\widehat{M}_\perp(\mathbf{p})$  are also shown on 3–dimensional plots in Figure 3, where the results on the top correspond to  $^{21}\text{Ne}$  and the results underneath correspond to  $^{37}\text{Ar}$ . As seen in this figure, the momentum distributions in  $^{37}\text{Ar}$  have a richer structure than those in  $^{21}\text{Ne}$ . This particularly rich intrinsic structure is characteristic of deformed odd–A nuclei with  $K = \frac{1}{2}$ . A characteristic of nuclei with  $K > \frac{1}{2}$  is the nodal line along the axis  $p_\perp = 0$  (see the top plots of Fig. 3). As a rule, the structure tends to become simpler for the higher  $K$  values. As an example, we show in Figure 4 the momentum distribution of  $^{25}\text{Mg}$ , that has  $K = \frac{5}{2}$ . In this case ( $\Omega_k = K = j$ ),  $\widehat{M}_z(\mathbf{p}) = n^{\text{odd}}(\mathbf{p})$  and  $\widehat{M}_\perp(\mathbf{p}) = 0$ . The function  $n^{\text{odd}}(\mathbf{p})$ , represented in the top of Fig. 4, shows a simple structure corresponding to a 99%  $d_{5/2}$  wave function. At the bottom of Fig. 4, one can also see the core momentum distribution in  $^{25}\text{Mg}$  which is very similar to that of  $^{21}\text{Ne}$ .

To understand the structure of these figures it is convenient to express the densities in momentum space in terms of the spherical components of the single–particle wave functions. Using Eqs. (9)–(12) one can show that

$$\begin{aligned} \overline{M}_\lambda(p) = n_\lambda(p) + n_\lambda^{\text{odd}}(p) &= \sum_i 2 \left[ v_i^2 + \delta_{i,k} \left( \frac{1}{2} - v_i^2 \right) \right] \sum_{\ell_j} \sum_{\ell'j'} \frac{1}{4\pi} \Theta_{i,\lambda}^{\ell_j, \ell'j'}(p) \\ &\times \left\{ \begin{matrix} j & j' & \lambda \\ \ell' & \ell & 1/2 \end{matrix} \right\} (-1)^{\Omega_i+1/2} \langle j \ \Omega_i \ j' \ -\Omega_i | \lambda \ 0 \rangle \end{aligned} \quad (24)$$

$$\widehat{M}_z(p_\perp, p_z) = \frac{\sqrt{6}}{4\pi} \sum_{L=\text{even}} \sum_{\lambda=\text{odd}} \hat{\lambda} \hat{L} P_L(\cos \theta_p) \begin{pmatrix} 1 & L & \lambda \\ 0 & 0 & 0 \end{pmatrix} \Pi_{L,\lambda}^0(p) \quad (25)$$

$$\widehat{M}_\perp(p_\perp, p_z) = -\frac{2\sqrt{3}}{4\pi} \sum_{L=\text{even}} \sum_{\lambda=\text{odd}} \frac{\hat{\lambda} \hat{L}}{\sqrt{(L+1)L}} P_L^1(\cos \theta_p) \begin{pmatrix} 1 & L & \lambda \\ 1 & -1 & 0 \end{pmatrix} \Pi_{L,\lambda}^0(p) \quad (26)$$

where we have introduced the momentum dependent density

$$\Pi_{L,\lambda}^0(p) = \sum_{\ell_j} \sum_{\ell'j'} \Theta_{k,L}^{\ell_j, \ell'j'}(p) \left\{ \begin{matrix} j' & \lambda & j \\ \ell' & L & \ell \\ 1/2 & 1 & 1/2 \end{matrix} \right\} (-1)^{\ell'+j'+K} \langle j \ K \ j' \ -K | \lambda \ 0 \rangle \quad (27)$$



with

$$\Theta_{k,L}^{\ell j, \ell' j'}(p) = \tilde{\phi}_k^{\ell j}(p) \tilde{\phi}_k^{\ell' j'}(p) \hat{\ell} \hat{\ell}' \hat{L} \hat{j} \hat{j}' \begin{pmatrix} \ell & \ell' & L \\ 0 & 0 & 0 \end{pmatrix} \quad (28)$$

We use the notation  $\hat{a} = \sqrt{2a+1}$ . Eq. (28) also applies to  $\Theta_{i,\lambda}^{\ell j, \ell' j'}(p)$  with  $k$  replaced by  $i$  and  $L$  by  $\lambda$ .  $\tilde{\phi}_k^{\ell j}(p)$  ( $\tilde{\phi}_i^{\ell j}(p)$ ) are the spherical components of the odd nucleon (core nucleons) wave function defined in Eqs. (12) and (13). Note that under interchange of  $p_z$  by  $-p_z$ , i.e., reflection through the plane perpendicular to the symmetry axis,  $\overline{M}(\mathbf{p})$  and  $\widehat{M}_z(\mathbf{p})$  are unchanged, whereas  $\widehat{M}_\perp(\mathbf{p})$  changes sign.

The fact that each individual single-particle wave function contains in general more than one  $\ell j$  component, makes it possible to have contributions to Eqs. (24)–(27) coming from several  $\ell j$ -components, as well as contributions from interferences between different  $\ell j$  waves. This is an important feature that would not be possible in the spherical limit, i.e., within the extreme spherical shell model. The lower the  $K$  value is, the lower  $\ell j$ -values are allowed in the single-particle wave functions, producing a richer structure in the momentum distributions (compare the results for  $^{25}\text{Mg}$ ,  $^{37}\text{Ar}$  and  $^{21}\text{Ne}$  in Figs. 1–4). In these sd-shell nuclei the dominant  $\ell j$  components of the outermost nucleons are  $\ell = 0$  and  $\ell = 2$ , though higher components are also present in our single-particle HF wave functions. In particular, the  $\ell = 0$  wave is allowed for the  $K = 1/2$  wave function while it is not for  $K = 3/2, 5/2$ , giving rise to the richer structure observed in  $^{37}\text{Ar}$ .

### III. Momentum Distribution Spin Matrix in Laboratory Frame. Polarization Considerations

To study spin dependent momentum distributions in laboratory frame we need to consider polarized nuclei. We consider a nucleus in its ground state with angular momentum  $J$  completely polarized in a given direction  $\mathbf{P}^*$  defined by the angles  $\Omega^* = (\theta^*, \phi^*)$  in the laboratory frame. The two by two matrix

$$M_{\sigma\sigma'}^{(J)}(\mathbf{p}, \Omega^*) = \langle JJ(\Omega^*) | a_{\mathbf{p}\sigma}^+ a_{\mathbf{p}\sigma'} | JJ(\Omega^*) \rangle \quad (29)$$

measures the probability to find a nucleon with momentum  $\mathbf{p}$  and spin projections  $\sigma, \sigma'$  in the laboratory frame. Our task here is to relate momentum distributions that may be observed in the laboratory to the intrinsic momentum distributions studied in the previous section. For this purpose it is also useful to consider partial contributions to Eq. (29) in addition to the total momentum distribution above defined. We note that Eq. (29) can also be written as

$$M_{\sigma\sigma'}^{(J)}(\mathbf{p}, \Omega^*) = \sum_R \langle JJ(\Omega^*) | a_{\mathbf{p}\sigma}^+ | R \rangle \langle R | a_{\mathbf{p}\sigma'} | JJ(\Omega^*) \rangle \quad (30)$$

where the sum is carried out over a complete set of states  $R$  of the residual system with  $A-1$  nucleons. In coincidence reactions of the type  $(x, x'N)$ , where  $x$  represents a leptonic or a hadronic projectile and  $N$  a knock-out nucleon detected in coincidence with the

scattered particle  $x'$ , one can select transitions to discrete states in the residual system and, therefore, one can measure partial momentum distributions of the form

$$M_{\sigma\sigma'}^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) = \sum_{M_R} \langle JJ(\Omega^*) | a_{\mathbf{p}\sigma}^+ | J_R M_R \rangle \langle J_R M_R | a_{\mathbf{p}\sigma'} | JJ(\Omega^*) \rangle. \quad (31)$$

In what follows we study the connection between the momentum distributions defined by Eqs. (29) and (31) and the intrinsic momentum distributions.

### III.1 Total Momentum Distributions

To compute the total spin dependent momentum distribution defined in Eq. (29) we first note that the magnetic substate  $|JJ(\Omega^*)\rangle$  must be expressed in terms of states referred to the same quantization axis as the spin components  $\sigma, \sigma'$ . We take this quantization axis to be the  $z$  axis of the laboratory fixed frame,

$$|JJ(\Omega^*)\rangle = \sum_M \mathcal{D}_{MJ}^J(\Omega^*) |JM\rangle \quad (32)$$

The convention for rotation operators and  $\mathcal{D}$  matrices is as in Refs.[22, 23].

On the other hand, we have to write the angular momentum eigenstates  $|JM\rangle$  in terms of the intrinsic ground state  $\Psi_k$  of the odd-A deformed nucleus. For that purpose we use the Bohr-Mottelson factorization approximation<sup>[24]</sup> and we get after some algebra

$$\begin{aligned} M_{\sigma\sigma'}^{(J)}(\mathbf{p}, \Omega^*) &= \sum_{\lambda\mu} \sqrt{4\pi\lambda} \begin{pmatrix} J & J & \lambda \\ J & -J & 0 \end{pmatrix} Y_\mu^\lambda(\Omega^*) \frac{\hat{J}^2}{16\pi^2} \int d\omega \sum_{\Sigma\Sigma'} \mathcal{D}_{\sigma\Sigma}^{1/2*}(\omega) \mathcal{D}_{\sigma'\Sigma'}^{1/2}(\omega) \\ &\times \left\{ \begin{pmatrix} J & J & \lambda \\ K & -K & 0 \end{pmatrix} (-1)^{J-K} \mathcal{D}_{\mu 0}^\lambda(\omega) \left[ \langle \Psi_k | a_{\mathbf{p}\Sigma}^+ a_{\mathbf{p}\Sigma'} | \Psi_k \rangle + (-1)^\lambda \langle \Psi_{\bar{k}} | a_{\mathbf{p}\Sigma}^+ a_{\mathbf{p}\Sigma'} | \Psi_{\bar{k}} \rangle \right] \right. \\ &+ \left. \begin{pmatrix} J & J & \lambda \\ K & K & -2K \end{pmatrix} \left[ \mathcal{D}_{\mu 2K}^\lambda(\omega) \langle \Psi_k | a_{\mathbf{p}\Sigma}^+ a_{\mathbf{p}\Sigma'} | \Psi_{\bar{k}} \rangle + (-1)^\lambda \mathcal{D}_{\mu -2K}^\lambda(\omega) \langle \Psi_{\bar{k}} | a_{\mathbf{p}\Sigma}^+ a_{\mathbf{p}\Sigma'} | \Psi_k \rangle \right] \right\} \quad (33) \end{aligned}$$

The same result is obtained in Projected HF to lowest order in  $\langle J^2 \rangle^{-1}$  (see Refs. [16, 22]). We note that the first two terms within the brackets are the intrinsic spin dependent momentum distributions  $M_{\Sigma\Sigma'}(\mathbf{p})$  and  $(-1)^{\Sigma-\Sigma'} M_{-\Sigma-\Sigma'}^*(\mathbf{p})$ , introduced in Eq. (14) (see also Eqs. (3) and (4)), whereas the two last terms are new. The latter depend only on the intrinsic momentum distributions  $\widehat{M}_z(p_z, p_\perp)$  and  $\widehat{M}_\perp(p_z, p_\perp)$ , which in turn depend only on the odd nucleon wave functions (see Eqs. (18) and (21)). We define  $\overline{M}^{(J)}(\mathbf{p}, \Omega^*)$  and  $\widehat{M}_\alpha^{(J)}(\mathbf{p}, \Omega^*)$  (with  $\alpha = 0, \pm 1$ ) as the trace and the spherical vector components, respectively, of the momentum distribution spin matrix in laboratory frame for the fully polarized nuclear ground state

$$\overline{M}^{(J)}(\mathbf{p}, \Omega^*) = \text{Tr} \left( M^{(J)}(\mathbf{p}, \Omega^*) \right) \quad (34)$$

$$\widehat{M}_\alpha^{(J)}(\mathbf{p}, \Omega^*) = \text{Tr} \left( M^{(J)}(\mathbf{p}, \Omega^*) \sigma_\alpha \right). \quad (35)$$

It is a simple matter to show that the scalar momentum distribution  $\overline{M}^{(J)}(\mathbf{p}, \Omega^*)$  is proportional to the intrinsic scalar momentum distribution and that the  $\alpha$  components of the vector momentum distribution depend only on the odd nucleon. Hence, also in the laboratory frame, the core nucleons contribute only to the trace  $\overline{M}^{(J)}(\mathbf{p}, \Omega^*)$  and not to  $\widehat{M}^{(J)}(\mathbf{p}, \Omega^*)$ . Taking the trace in Eq. (33) and using Eq. (22) it is easy to show that

$$\overline{M}^{(J)}(\mathbf{p}, \Omega^*) = \sum_{\lambda=\text{even}} \overline{M}_\lambda(p) P_\lambda(\cos \theta_p^*) G(\lambda; JK) \quad (36)$$

where  $\overline{M}_\lambda(p)$  are the multipoles of the intrinsic momentum distribution (see Eq. (24)),  $G(\lambda; JK)$  is a geometrical coefficient

$$G(\lambda; JK) = (-1)^{J-K} \hat{J}^2 \begin{pmatrix} J & J & \lambda \\ J & -J & 0 \end{pmatrix} \begin{pmatrix} J & J & \lambda \\ K & -K & 0 \end{pmatrix}, \quad (37)$$

and  $\theta_p^*$  is the relative angle between the momentum  $\mathbf{p}$  and the polarization direction  $\mathbf{P}^*$ .

The expression for  $\widehat{M}^{(J)}(\mathbf{p}, \Omega^*)$  is somewhat more involved, as one gets contributions from all the terms in Eq. (33) that correspond to different angular momentum projections of the intrinsic odd nucleon densities  $\widehat{M}(\mathbf{p})$ . After a lengthy but straightforward algebra, using Eqs. (3) and (4), we get

$$\widehat{M}_\alpha^{(J)}(\mathbf{p}, \Omega^*) = \sqrt{6} \sum_{L=\text{even}} \sum_{\lambda=\text{odd}} \sum_{\mu} (-1)^\alpha Y_\mu^\lambda(\Omega^*) Y_{\alpha-\mu}^L(\Omega_p) \begin{pmatrix} 1 & L & \lambda \\ \alpha & \mu - \alpha & -\mu \end{pmatrix} \mathcal{L}_{L,\lambda}^{JK}(p) \quad (38)$$

with

$$\mathcal{L}_{L,\lambda}^{JK}(p) = G(\lambda; JK) \left\{ \Pi_{L,\lambda}^0(p) + (-1)^{J+K} \frac{\langle J K J K | \lambda 2K \rangle}{\langle J K J -K | \lambda 0 \rangle} \Pi_{L,\lambda}^{2K}(p) \right\} \quad (39)$$

and  $\alpha = 0, \pm 1$ . The momentum dependent densities are given by

$$\Pi_{L,\lambda}^{2K}(p) = \sum_{\ell_j} \sum_{\ell'_j} \Theta_{k,L}^{\ell_j, \ell'_j}(p) \begin{Bmatrix} j' & \lambda & j \\ \ell' & L & \ell \\ 1/2 & 1 & 1/2 \end{Bmatrix} (-1)^{\ell'} \langle j K j' K | \lambda 2K \rangle \quad (40)$$

and  $\Pi_{L,\lambda}^0(p)$  as given in Eq. (27). Note that the signature dependent term,  $\Pi_{L,\lambda}^{2K}(p)$ , only contributes for  $\lambda$ -values satisfying  $2K \leq \lambda \leq 2J$ , i.e.,  $\lambda = 2J$  when  $K = J$ .

If we fix the polarization direction as the  $z$ -axis in laboratory frame, Eq. (38) reduces to a simpler angular dependence involving the direction of  $\mathbf{p}$  only:

$$\widehat{M}_\alpha^{(J)}(\mathbf{p}, \Omega^* = 0) = \sqrt{6} \sum_{L=\text{even}} \sum_{\lambda=\text{odd}} (-1)^\alpha \frac{\hat{\lambda}}{\sqrt{4\pi}} Y_\alpha^L(\Omega_p^*) \begin{pmatrix} 1 & L & \lambda \\ \alpha & -\alpha & 0 \end{pmatrix} \mathcal{L}_{L,\lambda}^{JK}(p). \quad (41)$$

We then define  $\widehat{M}_l^{(J)}$ ,  $\widehat{M}_s^{(J)}$  and  $\widehat{M}_n^{(J)}$  as the vector momentum distribution components along the polarization axis ( $l$ ) and in the transverse directions ( $s, n$ ). Denoting by  $(\theta_p^*, \varphi_p^*)$  the direction of  $\mathbf{p}$  in laboratory frame with the  $z$ -axis parallel to  $\mathbf{P}^*$  we have

$$\widehat{M}_l^{(J)}(\mathbf{p}, \Omega^* = 0) = \frac{\sqrt{6}}{4\pi} \sum_{L=\text{even}} \sum_{\lambda=\text{odd}} \hat{\lambda} \hat{L} P_L(\cos \theta_p^*) \begin{pmatrix} 1 & L & \lambda \\ 0 & 0 & 0 \end{pmatrix} \mathcal{L}_{L,\lambda}^{JK}(p) \quad (42)$$

$$\widehat{M}_s^{(J)}(\mathbf{p}, \Omega^* = 0) = \frac{-1}{\sqrt{2}} \left( \widehat{M}_1^{(J)} - \widehat{M}_{-1}^{(J)} \right) \equiv \cos \varphi_p^* \widehat{M}_t^{(J)}(\mathbf{p}, \Omega^* = 0) \quad (43)$$

$$\widehat{M}_n^{(J)}(\mathbf{p}, \Omega^* = 0) = \frac{i}{\sqrt{2}} \left( \widehat{M}_1^{(J)} + \widehat{M}_{-1}^{(J)} \right) \equiv \sin \varphi_p^* \widehat{M}_t^{(J)}(\mathbf{p}, \Omega^* = 0) \quad (44)$$

with the transverse component defined as

$$\widehat{M}_t^{(J)}(\mathbf{p}, \Omega^* = 0) = -\frac{2\sqrt{3}}{4\pi} \sum_{L=\text{even}} \sum_{\lambda=\text{odd}} \frac{\hat{\lambda}\hat{L}}{\sqrt{(L+1)L}} P_L^1(\cos \theta_p^*) \begin{pmatrix} 1 & L & \lambda \\ 1 & -1 & 0 \end{pmatrix} \mathcal{L}_{L,\lambda}^{JK}(p) . \quad (45)$$

The momentum distribution in the laboratory frame for the unpolarized nucleus can be easily obtained by integrating Eqs. (36) and (38) over the polarization direction. Clearly for an **unpolarized** nucleus only the  $\lambda = 0$  multipole of the scalar momentum distribution remains:

$$\int d\Omega^* \widehat{M}_\alpha^{(J)}(\mathbf{p}, \Omega^*) = 0 , \quad \int d\Omega^* \overline{M}^{(J)}(\mathbf{p}, \Omega^*) = 4\pi \overline{M}_{\lambda=0}(p) \quad (46)$$

For  $\lambda = 0$ , Eq. (24) reduces to

$$\overline{M}_{\lambda=0}(p) = n_{\lambda=0}(p) + n_{\lambda=0}^{\text{odd}}(p) = \frac{1}{4\pi} \sum_i 2 \left[ v_i^2 + \delta_{i,k} \left( \frac{1}{2} - v_i^2 \right) \right] \sum_{\ell_j} |\tilde{\phi}_i^{\ell_j}(p)|^2 . \quad (47)$$

Hence in the **unpolarized** case one loses not only all of the information contained in the vector momentum distribution but also important information contained in the scalar momentum distribution. One misses the  $\lambda \geq 2$  multipoles of the core and odd nucleon momentum densities that, as discussed in Section II, carry important information on shell admixtures and on details of the internal dynamics which are crucial to attain equilibrium in open shell nuclei.

Now we can easily compare the scalar momentum distribution in laboratory and intrinsic frames. Comparison of Eq. (22) to Eq. (36) shows that the dependence on the direction of  $\mathbf{p}$  relative to the intrinsic symmetry axis, is replaced in the laboratory frame by a similar dependence on the direction of  $\mathbf{p}$  relative to the polarization direction, weighted by a geometrical coefficient  $G(\lambda; JK)$ . This geometrical coefficient takes the value 1 for  $\lambda = 0$  and decreases as  $\lambda$  increases ( $G(\lambda; JK) = 0$  for  $\lambda > 2J$ ). For  $^{37}\text{Ar}$  and  $^{21}\text{Ne}$  the only non-zero coefficient with  $\lambda > 0$  is  $G(\lambda = 2) = 1/5, -1/5$ , respectively.

For the vector momentum distribution we also see that the expressions for longitudinal and transverse components in the laboratory frame involve the intrinsic multipoles  $\Pi_{L,\lambda}^0(p)$ . These expressions are similar to the ones in the intrinsic frame (compare Eqs. (42) and (45) to Eqs. (25) and (26)), replacing the direction of  $\mathbf{p}$  relative to the internal symmetry axis by the direction of  $\mathbf{p}$  relative to the polarization direction, and weighting the intrinsic  $\lambda$  multipoles with the geometrical factor  $G(\lambda; JK)$ . However in the case of the vector components one has additional contributions for the  $\lambda \geq 2K$  multipoles coming from the signature dependent terms (see Eqs. (39) and (40)).

### III.2 Partial Momentum Distributions

In this section we consider partial contributions, as defined in Eq. (31), to the spin dependent momentum distribution previously discussed. These partial contributions play a central role in coincidence  $\vec{A}(\vec{x}, x'N)R$  experiments where transitions to discrete states  $J_R$  of the residual nucleus are selected. In these processes the dependence on the nuclear structure of the cross section contains as basic ingredients the spin dependent transition densities in momentum space  $M_{\sigma\sigma'}^{(J \rightarrow J_R)}$  defined in Eq. (31) (see Refs. [11, 25]).

As seen in previous sections, where we considered total momentum distributions, only the unpaired nucleon contributes to the vector momentum distribution. On the contrary, the scalar momentum distribution receives contribution from the core nucleons as well as from the unpaired nucleon, but the latter depends more strongly on the specific structure of the particular nucleus under consideration. In the contribution from the core, the  $\lambda = 0$  multipole (occurring also for unpolarized nuclei) dominates, while in the contribution from the odd nucleon the multipoles with  $\lambda > 0$  are comparable to the  $\lambda = 0$  multipole. Thus, it is interesting to focus on transitions in which the knock-out nucleon is the unpaired one, and the intrinsic structure of the residual nucleus is basically given by that of the even-even core of the parent nucleus. This is the case for transitions to the low lying states in the residual nucleus which are populated when the odd nucleon is knocked out.

We consider a transition from a 100% polarized nucleus in the direction  $\mathbf{P}^* = (\Omega^*)$  to a discrete state  $J_R$  in the residual nucleus of unobserved polarization. The spin dependent transition density in momentum space can then be written as

$$M_{\sigma\sigma'}^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) = \sum_{\lambda\mu} \hat{\lambda}^2 \mathcal{D}_{\mu 0}^{\lambda*}(\Omega^*) \begin{pmatrix} J & J & \lambda \\ -J & J & 0 \end{pmatrix} \\ \times \sum_{MM'M_R} (-1)^{J-M} \begin{pmatrix} J & J & \lambda \\ -M & M' & \mu \end{pmatrix} \langle JM | a_{\mathbf{p}\sigma}^+ | J_R M_R \rangle \langle JM' | a_{\mathbf{p}\sigma'}^+ | J_R M_R \rangle^*, \quad (48)$$

where we have replaced Eq. (32) into Eq. (31) and used composition of rotation matrices. In addition here we specialize to states  $J_R$  in the ground state rotational band ( $K^\pi = 0^+$ ) of the residual nucleus. The matrix elements entering into Eq. (48) are in this case

$$\langle JM | a_{\mathbf{p}\sigma}^+ | J_R M_R \rangle = \frac{\hat{J} \hat{J}_R}{8\pi^2 \sqrt{2}} \sum_{\Sigma} \int d\omega \mathcal{D}_{MR0}^{J_R*}(\omega) \mathcal{D}_{\sigma\Sigma}^{1/2*}(\omega) \\ \times \left\{ \mathcal{D}_{MK}^J(\omega) \langle \Phi_k | a_{\mathbf{p}\Sigma}^+ | \Phi_0 \rangle + (-1)^{J-K} \mathcal{D}_{M-K}^J(\omega) \langle \Phi_{\bar{k}} | a_{\mathbf{p}\Sigma}^+ | \Phi_0 \rangle \right\} \quad (49)$$

with  $\Phi_0$  the intrinsic ground state of the even-even core.

Using the  $\ell j$ -wave expansion of the single-particle wave functions in the intrinsic frame (see Eqs. (9) and (10)), and transforming to the laboratory frame we find that

$$\langle \Phi_k | a_{\mathbf{p}\Sigma}^+ | \Phi_0 \rangle = \langle \chi_{\Sigma} | \tilde{\Phi}_k(\mathbf{p}, s) \rangle^* = \sum_{\ell j} \sum_{m_{\ell}} \tilde{\phi}_k^{\ell j}(\mathbf{p}) \langle \ell \ K - \Sigma \ \frac{1}{2} \ \Sigma | j \ K \rangle Y_{m_{\ell}}^{\ell*}(\Omega_{\mathbf{p}}) \mathcal{D}_{m_{\ell} \ K - \Sigma}^{\ell*}(\omega). \quad (50)$$

A similar expression is obtained for  $\langle \Phi_{\bar{k}} | a_{\mathbf{p}\Sigma}^+ | \Phi_0 \rangle$  replacing  $K$  by  $-K$  and multiplying by a factor  $(-1)^{j-K}$  in Eq. (50).

Upon substitution of Eq. (50) into Eq. (49) and integration over the direction of the intrinsic frame we get

$$\begin{aligned} \langle JM | a_{\mathbf{p}\sigma}^+ | J_R M_R \rangle &= \hat{J} \hat{J}_R \frac{(1 + (-1)^{J_R})}{\sqrt{2}} \sum_{\ell j} \sum_{m_\ell \mu} \tilde{\phi}_k^{\ell j}(p) \begin{pmatrix} J_R & J & J \\ 0 & K & -K \end{pmatrix} \\ &\times Y_{m_\ell}^{\ell*}(\Omega_p) \langle \ell \ m_\ell \ \frac{1}{2} \ \sigma | j \ \mu \rangle \begin{pmatrix} J_R & J & j \\ -M_R & M & -\mu \end{pmatrix} (-1)^{K-M} \end{aligned} \quad (51)$$

The factor  $(1 + (-1)^{J_R})$  appears from the sum of the two contributions in Eq. (49), and keeps track of the well known property that  $0^+$  bands only contain states with even  $J$  values, as is the case for the ground state bands of even-even axially symmetric nuclei with reflexion symmetry through a plane perpendicular to the symmetry axis.

Defining scalar and vector transition densities in analogy to the scalar and vector total momentum distributions,

$$\overline{M}^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) = \text{Tr} \left( M^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) \right) \quad (52)$$

$$\widehat{M}_\alpha^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) = \text{Tr} \left( M^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) \sigma_\alpha \right), \quad (53)$$

we readily find after substitution of Eq. (51) into Eq. (48) and computation of the traces in spin space:

$$\begin{aligned} \overline{M}^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) &= \sum_{\lambda=\text{even}} \frac{\hat{\lambda}}{4\pi} P_\lambda(\cos \theta_p^*) \begin{pmatrix} J & J & \lambda \\ -J & J & 0 \end{pmatrix} \hat{J}^2 (-1)^{J-K} \\ &\times \sum_{\ell j} \sum_{\ell' j'} \Theta_{k,\lambda}^{\ell j, \ell' j'}(p) \begin{Bmatrix} j' & j & \lambda \\ \ell & \ell' & 1/2 \end{Bmatrix} (-1)^{K+j+j'-1/2} \\ &\times \begin{pmatrix} J_R & J & j \\ 0 & K & -K \end{pmatrix} \begin{pmatrix} J_R & J & j' \\ 0 & K & -K \end{pmatrix} \begin{Bmatrix} j' & j & \lambda \\ J & J & J_R \end{Bmatrix} \hat{J}_R^2 (1 + (-1)^{J_R}) \end{aligned} \quad (54)$$

$$\begin{aligned} \widehat{M}_\alpha^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) &= \sqrt{6} \sum_{L=\text{even}} \sum_{\lambda=L \pm 1} \sum_{\mu} (-1)^\alpha Y_\mu^\lambda(\Omega^*) Y_{\alpha-\mu}^L(\Omega_p) \\ &\times \begin{pmatrix} 1 & L & \lambda \\ \alpha & \mu - \alpha & -\mu \end{pmatrix} (-1)^{J-K} \begin{pmatrix} J & J & \lambda \\ -J & J & 0 \end{pmatrix} \hat{J}^2 \hat{\lambda} \\ &\times \sum_{\ell j} \sum_{\ell' j'} \Theta_{k,L}^{\ell j, \ell' j'}(p) \begin{Bmatrix} L & \lambda & 1 \\ \ell' & j' & 1/2 \end{Bmatrix} (-1)^{\ell'+j'+K} (-1)^{j-j'} \\ &\times \begin{pmatrix} J_R & J & j \\ 0 & K & -K \end{pmatrix} \begin{pmatrix} J_R & J & j' \\ 0 & K & -K \end{pmatrix} \begin{Bmatrix} j' & j & \lambda \\ J & J & J_R \end{Bmatrix} \hat{J}_R^2 (1 + (-1)^{J_R}). \end{aligned} \quad (55)$$

Note that these scalar and vector transition densities are made up of the same building blocks (the odd nucleon densities  $\Theta_{k,L}^{\ell j, \ell' j'}(p)$  defined in Eq. (28)) as the intrinsic and laboratory scalar and vector momentum distributions, respectively, and have similar expressions.

The only difference is that for each  $\lambda$  multipole the sums over  $\ell j, \ell' j'$  are now restricted to angular momentum values satisfying  $|J - J_R| \leq j, j' \leq J + J_R$ ;  $|j - J_R| \leq j' \leq j + J_R$ , while in Eqs. (24)–(27) and (36)–(40) the sums over  $\ell j, \ell' j'$  do not have these restrictions.

Furthermore, using the relation

$$\begin{aligned} & \sum_{J_R} \hat{J}_R^2 \hat{\lambda}^2 \left(1 + (-1)^{J_R}\right) \begin{pmatrix} J_R & J & j \\ 0 & K & -K \end{pmatrix} \begin{pmatrix} J_R & J & j' \\ 0 & K & -K \end{pmatrix} \begin{Bmatrix} j' & j & \lambda \\ J & J & J_R \end{Bmatrix} \\ &= (-1)^{j-j'} \left[ (-1)^{2J+\lambda} \langle j \ K \ j' \ -K | \lambda \ 0 \rangle \langle J \ K \ J \ -K | \lambda \ 0 \rangle \right. \\ & \times \left. \delta_{\lambda, \text{odd}} (-1)^{J+j'+2K} \langle j \ K \ j' \ K | \lambda \ 2K \rangle \langle J \ K \ J \ K | \lambda \ 2K \rangle \right] \end{aligned} \quad (56)$$

that we have derived from the relations in Appendix II of Ref. [23], one can show that

$$\sum_{J_R} \overline{M}^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) = \sum_{\lambda=\text{even}} P_\lambda(\cos \theta_p^*) n_\lambda^{\text{odd}}(p) G(\lambda; JK), \quad (57)$$

which is the odd nucleon contribution to the total scalar momentum distribution  $\overline{M}^{(J)}(\mathbf{p}, \Omega^*)$  (see Eq. (36)), and that

$$\sum_{J_R} \widehat{M}_\alpha^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*) = \widehat{M}_\alpha^{(J)}(\mathbf{p}, \Omega^*) \quad (58)$$

We would like to remark that in Eqs. (57) and (58) the sum over  $J_R$  runs over states belonging to the ground state band in the residual nucleus. Therefore, these equations tell that measuring the transition densities to each state in the ground state band one can map out entirely the intrinsic momentum distribution spin matrix of the odd nucleon. In addition, Eq. (58) tells us that all possible information contained in the vector momentum distribution of the polarized target nucleus can be obtained by measuring the transition densities to the ground state band in the residual nucleus. The same is true for the odd nucleon contribution to the scalar momentum distribution (see Eq. (57)), which as noted above is more interesting than the core contribution in the sense that it depends more on the nuclear structure and on the polarization. Obviously, transitions to higher excited states will bring information on momentum distributions in two quasiparticle excitations, vibrational excitations,..., but will not add additional information on the spin dependent momentum distribution of the polarized nuclear target.

As already mentioned, the only difference between the partial and total momentum distributions in laboratory frame is the restriction imposed by the  $J_R$  value. This restriction however, may cause the transition densities to have quite different structures depending on the  $J_R$  value and on the particular nucleus considered. This is shown in Figures 5 and 6 where we represent  $\overline{M}^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*)$ ,  $\widehat{M}_l^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^* = 0)$ , and  $\widehat{M}_t^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^* = 0)$  for  $J_R = 0$  and  $J_R = 2$ , respectively. The longitudinal ( $\widehat{M}_l^{(J \rightarrow J_R)}$ ) and transverse ( $\widehat{M}_t^{(J \rightarrow J_R)}$ ) components (with respect to the polarization direction) of the vector transition densities are defined in analogy to  $\widehat{M}_l^{(J)}$  and  $\widehat{M}_t^{(J)}$  in Eqs. (42)–(45). Fig. 5 is for the transition from the ground state in  $^{21}\text{Ne}$  to the ground state in  $^{20}\text{Ne}$ . The scalar transition density is shown on top, and the longitudinal and transverse components of the vector transition

density are shown below. For this transition ( $J \rightarrow J_R = 0$ ) the momentum dependence of the transition densities does not change appreciably in going from  $^{21}\text{Ne}$  to  $^{37}\text{Ar}$ , only their strength change. Thus, Fig. 5 also serves to represent the transition densities for the ground state in  $^{37}\text{Ar}$  to the ground state in  $^{36}\text{Ar}$ . For this purpose one has to multiply the results shown in Fig. 5 by a factor  $\approx 14$ , which is the ratio between the  $d_{3/2}$  strength in the odd nucleon wave functions of  $^{37}\text{Ar}$  and  $^{21}\text{Ne}$ .

On the contrary, the momentum dependence changes much in going from  $^{21}\text{Ne}$  to  $^{37}\text{Ar}$  when considering the transition to the first  $2^+$  states of their respective residual nuclei ( $^{20}\text{Ne}$  and  $^{36}\text{Ar}$ ). This is clearly seen in Fig. 6, where the plots on the left correspond to  $^{21}\text{Ne} (3/2^+) \rightarrow ^{20}\text{Ne} (2_1^+)$  and the plots on the right correspond to  $^{37}\text{Ar} (3/2^+) \rightarrow ^{36}\text{Ar} (2_1^+)$ . Scalar momentum distributions are shown on the top and longitudinal ( $l$ ) and transverse ( $t$ ) components of the vector momentum distributions for these transitions are shown below. As already noted when comparing the intrinsic momentum distribution for  $^{21}\text{Ne}$  and  $^{37}\text{Ar}$  in section II, in the case of  $^{37}\text{Ar}$  the odd nucleon wave function contains mainly a mixture of  $s$  and  $d$  waves, whereas in the case of  $^{21}\text{Ne}$  it contains mainly  $d$  wave. For the transitions  $J \rightarrow J_R = 0$  only the  $d_{3/2}$  wave component contributes in either case, thus resulting in similar momentum dependence of the transition densities. For the transitions  $J \rightarrow J_R = 2$  all positive parity waves with  $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$  may contribute, but the  $j = \frac{1}{2}$  is only present in the odd nucleon of  $^{37}\text{Ar}$ , not in  $^{21}\text{Ne}$ . This, together with the fact that also the  $\frac{3}{2}, \frac{5}{2}, \frac{7}{2}$  components enter with different amplitudes in each case, explains why the momentum dependences in Fig. 6 are different for  $^{21}\text{Ne}$  and  $^{37}\text{Ar}$ . Finally we would like to remark that the transition densities shown in Figs. 5 and 6 are the dominant contributions to the total scalar and vector momentum distributions in  $^{21}\text{Ne}$  and  $^{37}\text{Ar}$ . Transitions densities to higher excited states  $4^+, 6^+, \dots$  in the ground state band of the residual nuclei are much smaller than the ones seen in Figs. 5 and 6.

### III.3 Comparison with Spherical Case

It is interesting to compare the results in previous sections with the ones corresponding to spherical nuclei. The case of spherical nuclei can be thought of as a limiting case when the deformation of the mean field goes to zero. In this limit the single particle wave functions are eigenstates of angular momentum and the system does not have a preferred direction unless the nucleus is polarized by an external field. Hence the main two differences with the deformed case are that now, i) each single particle wave function has a single  $\ell j$ -wave component ( $\tilde{\phi}_{\ell j}^i(p) = \delta_{\ell, \ell_i} \delta_{j, j_i} \tilde{R}_{n_i \ell_i j_i}(p)$ ), and ii) there is no distinction between intrinsic and laboratory frames. We denote by  $\{i\} \equiv \{n_i \ell_i j_i\}$  the set of orbitals filled by the nucleons of the even-even core and by  $\{k\} \equiv \{n_k \ell_k j_k\}$  the odd nucleon orbital, which has  $j_k = J$  and  $\ell_k$  fixed by the parity and  $J$  values.

The scalar momentum distribution for the fully polarized nucleus is then given by

$$\overline{M}^{(J)\text{sph.}}(\mathbf{p}, \Omega^*) = \sum_{\lambda=\text{even}} P_{\lambda}(\cos \theta_p^*) (\delta_{\lambda, 0} n^{\text{sph.}}(p) + n_{\lambda}^{\text{odd, sph.}}(p)) \quad (59)$$



where “sph.” stands for spherical limit; the odd nucleon multipoles are given by

$$n_{\lambda}^{\text{odd,sph.}}(p) = \frac{\hat{\ell}_k^2 \hat{\lambda}^2 \hat{j}_k^2}{4\pi} |\tilde{R}_{n_k \ell_k j_k}(p)|^2 \begin{pmatrix} \ell_k & \ell_k & \lambda \\ 0 & 0 & 0 \end{pmatrix} (-1)^{j_k + \frac{1}{2}} \begin{pmatrix} j_k & j_k & \lambda \\ j_k & -j_k & 0 \end{pmatrix} \begin{Bmatrix} j_k & j_k & \lambda \\ \ell_k & \ell_k & \frac{1}{2} \end{Bmatrix} \delta_{j_k, J} \quad (60)$$

and  $n^{\text{sph.}}(p)$  is the spherically symmetric momentum distribution of the core

$$n^{\text{sph.}}(p) = n_0(p) = \frac{1}{4\pi} \sum_i v_i^2 (2j_i + 1) |\tilde{R}_{n_i \ell_i j_i}(p)|^2. \quad (61)$$

A thorough study of the properties of  $n(p)$  for even–even spherical nuclei can be found in Refs. [26, 27]. The differences between scalar momentum distributions in the spherical and deformed case are now made explicit by comparing Eqs. (59)–(61) with Eq. (36) (see also Eq. (24)). The core has now only a monopole contribution, while in the deformed case the core has also small multipoles  $n_{\lambda}(p)$  with  $\lambda > 0$ , that approximately average to zero after integration on  $p$  (see Fig. 2 and table II).

For the odd nucleon contribution the sum over  $\ell j, \ell' j'$  in Eq. (27) is now reduced to a single term  $\ell j = \ell' j' = \ell_k j_k$ , with  $\ell_k j_k$  the values corresponding to the spin and parity of the nucleus, and with  $\Omega_k = K = j_k = J$ . Therefore, in the spherical case all the  $\lambda$ –multipoles have the same  $p$  dependence, whereas in the deformed case for each  $\lambda$  value we may have a different dependence on  $p$ . The geometrical coefficient  $G(\lambda; JK)$ , which appears in the deformed case from the transformation between intrinsic and laboratory frames, is not present in the spherical case. This reflects the fact that now, “a priori”, there is no internal preferred direction.

The longitudinal ( $l$ ) and transverse ( $t$ ) components of the vector momentum distributions are

$$\begin{aligned} \widehat{M}_{\left\{ \begin{smallmatrix} l \\ t \end{smallmatrix} \right\}}^{(J)\text{sph.}}(\mathbf{p}, \Omega^* = 0) &= |\tilde{R}_{n_k \ell_k j_k}(p)|^2 \frac{\hat{\ell}_k^2 \hat{j}_k^2 \sqrt{6}}{4\pi} \delta_{j_k, J} \sum_{L=\text{even}} \sum_{\lambda=L\pm 1} \hat{\lambda}^2 \hat{L}^2 \begin{pmatrix} \ell_k & \ell_k & L \\ 0 & 0 & 0 \end{pmatrix} \\ &\times (-1)^{\ell_k + 1} \begin{pmatrix} j_k & j_k & \lambda \\ j_k & -j_k & 0 \end{pmatrix} \begin{Bmatrix} L & \lambda & 1 \\ \ell_k & j_k & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} L & \lambda & 1 \\ \ell_k & j_k & \frac{1}{2} \end{Bmatrix} \left\{ \begin{array}{l} P_L(\cos \theta_p^*) \begin{pmatrix} 1 & L & \lambda \\ 0 & 0 & 0 \end{pmatrix} \\ \frac{-P_L^1(\cos \theta_p^*)}{\sqrt{\frac{L(L+1)}{2}}} \begin{pmatrix} 1 & L & \lambda \\ 1 & -1 & 0 \end{pmatrix} \end{array} \right\} \quad (62) \end{aligned}$$

where for simplicity we have chosen the polarization direction along the laboratory  $z$  axis. Again, by comparing Eq. (62) to Eqs. (42) and (45) (see also Eqs. (39) and (40)) we see that the differences with the deformed case are the absence of the geometrical factor  $G(\lambda; JK)$  and the restriction of the sum over  $\ell j, \ell' j'$  to a single term with  $\ell_k j_k$  dictated by the spin and parity of the nucleus, and with  $\Omega_k = j_k = J$ . It is important to remark that this restriction produces equal  $p$ –dependence of the various scalar and vector momentum multipoles.

It is also interesting to remark the similarity between Eq. (62), that gives the longitudinal and transverse vector momentum distributions with respect to the polarization direction, and Eqs. (25) and (26), that give the intrinsic vector momentum distribution of the deformed nucleus along the symmetry axis and in the perpendicular plane. If one

restricts Eq. (27) to a single term with  $\ell j = \ell' j' = \ell_k j_k$ , and takes  $\Omega_k = j_k = J$ , the above mentioned expressions are identical to Eq. (62). This reflects the fact that for spherical nuclei polarizing the nucleus is the only way to define an internal preferred direction for the system.

Expressions for transition densities in this case can also be easily derived. Obviously in this case to map out the vector momentum distribution and the odd nucleon contribution to the scalar momentum distribution one needs only to consider the transition in which the residual nucleus is left in its ground state ( $J_R = 0$ ). It is easy to show that in this case the vector momentum distribution for the transition  $J \rightarrow J_R = 0$  is also given by Eq. (62), and the scalar momentum distribution is given by the odd nucleon contribution in Eq. (59). For the case of a spherical nucleus with odd nucleon in a single  $d_{\frac{5}{2}}$  orbital these momentum distributions have the same structure as the one shown in the top plot of Fig. 4.

## IV. Summary and Final Remarks

We have studied spin dependent momentum distributions in deformed nuclei on the basis of the selfconsistent mean field approximation. The spin degree of freedom is relevant for odd-A nuclei that can be polarized. We consider the case of axially and reflection symmetric mean fields and discuss momentum distributions in the nuclear ground state. We first study spin dependent momentum distributions in the intrinsic frame for a set of deformed nuclei in the sd-shell ( $^{21}\text{Ne}$ ,  $^{37}\text{Ar}$ ,  $^{25}\text{Mg}$ ), and then relate them to momentum distributions in the laboratory frame and to transition densities in momentum space that can in principle be measured in one nucleon knock-out reactions. We do this in a systematic way by decomposing the spin dependent two by two matrix  $M_{\sigma\sigma'}(\mathbf{p})$  into scalar and vector components in spin space. We show that the scalar and vector momentum distributions that can be measured in the laboratory are intimately related to the corresponding scalar and vector intrinsic momentum distributions. Thus, measuring the former gives information on the latter.

The study carried out for the selected nuclei ( $^{21}\text{Ne}$ ,  $^{37}\text{Ar}$ ,  $^{25}\text{Mg}$ ) shows that the even-even cores, which contribute only to the scalar momentum distribution, are dominated by the  $\lambda = 0$  multipole but contain also quadrupole (and higher) multipoles that are small and tend to average to zero upon integration on  $p$ . Actually, this isotropy condition ( $\langle p_{\perp}^2 \rangle = \langle p_z^2 \rangle$ ) that was already discussed for the case of even-even nuclei in Refs. [8, 9], is found here to be also approximately satisfied for the odd-A deformed nuclei. More involved structure is found in the vector momentum distribution and, to a lesser extent, in the odd nucleon contribution to the scalar momentum distribution.

The vector momentum distributions are found to have very rich structures, particularly for  $^{37}\text{Ar}$  which is the nucleus with lowest  $K$  value in its ground state. The richness in structure of the vector momentum distribution, as well as of the odd nucleon contribution to the scalar momentum distribution, is found to decrease with increasing  $K$  values. Information on this internal structure can be gained by measuring the transition densities in momentum space by one nucleon knock-out reactions from a **polarized** target. It is also shown that most of the interesting information can be obtained from transitions to

the lowest states (ground state band) in the residual nucleus. An application to  $\vec{A}(\vec{e}, e'n)B$  reaction on  $^{21}\text{Ne}$  has already been made<sup>[11]</sup> and applications to the same type of reaction on  $^{37}\text{Ar}$  and  $^{37}\text{K}$  are now under study<sup>[17]</sup>. The expressions relating intrinsic and laboratory momentum distributions have been derived here within a similar philosophy to that in Refs. [16, 28] concerning the relations between intrinsic and laboratory multipoles measured by  $(e, e')$  reactions.

We have also compared the results on deformed nuclei with results for spherical nuclei. This comparison shows that in the spherical limit the structure of vector and scalar momentum distributions is much simpler, even when we consider a polarized odd- $A$  nucleus. Obviously for unpolarized nuclei, whether spherical or deformed, only the monopole part of the scalar momentum distribution enters into the picture; neither the vector momentum distribution nor the higher multipoles of the scalar momentum distributions are accessible in unpolarized nuclei.

The results presented have been obtained with the SKA interaction, similar results are obtained with other Skyrme type forces. It would be interesting to check whether the results presented here are modified when finite range effective interactions, like Gogny force<sup>[29]</sup>, are used. In principle this would allow to study effects of short-range dynamical correlations not included in the present calculations. It would also be interesting to study the results obtained with relativistic deformed HF calculations. The latter calculations may be superior to the non-relativistic ones for the study of spin degrees of freedom. In particular for odd- $A$  nuclei the relativistic formalism allows to take into account in a simpler way effects due to time reversal non invariant terms in the mean field that have not been considered in the present work. In the future we plan to study these points.

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## Figure Captions

Figure 1: Contributions from the even–even core (left) and from the odd nucleon (right) to the scalar momentum distribution in  $^{21}\text{Ne}$  (top) and  $^{37}\text{Ar}$  (bottom).

Figure 2: Monopole ( $n_0$ ) and quadrupole ( $n_2$ ) contributions (in  $\text{fm}^3$ ) to the even–even core momentum distributions (normalized to 1) of  $^{21}\text{Ne}$  and  $^{37}\text{Ar}$  from DDHF calculations and from Nilsson model calculations. The results obtained with the Nilsson model without major shell admixtures are labelled with primes.

Figure 3: Components of the vector momentum distribution parallel and perpendicular to the symmetry axis for  $^{21}\text{Ne}$  (top) and  $^{37}\text{Ar}$  (bottom).

Figure 4: Core (bottom) and odd nucleon (top) momentum distributions in  $^{25}\text{Mg}$ . The vector momentum distribution has a single component parallel to the symmetry axis that coincide with the odd nucleon contribution to the scalar momentum distribution ( $\widehat{M}_z(\mathbf{p}) = n^{\text{odd}}(\mathbf{p})$ ).

Figure 5: Scalar ( $\overline{M}^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^*)$ ) and vector components ( $\widehat{M}_{l,t}^{(J \rightarrow J_R)}(\mathbf{p}, \Omega^* = 0)$ ) of the transitions densities in momentum space (in  $\text{fm}^3$ ) for  $J \rightarrow J_R = 0$  in  $^{21}\text{Ne}$ . The results for  $^{37}\text{Ar}$  are similar except for a scale factor  $\sim 14$ .

Figure 6: Same as Fig. 5 for the transition  $J \rightarrow J_R = 2$  in  $^{21}\text{Ne}$  (left) and in  $^{37}\text{Ar}$  (right).

Table I: Results of DDHF calculations for binding energies, proton and mass quadrupole moments, r.m.s. radii, moments of inertia and decoupling parameters.

Nucleus	$B$ (MeV)	$Q^\pi$ (fm <sup>2</sup> )	$Q^M$ (fm <sup>2</sup> )	$\langle r^2 \rangle^{\frac{1}{2}}$ (fm)	$\mathcal{I}$ (MeV <sup>-1</sup> )	$a$
<sup>21</sup> Ne	-165.1	44.0	92.1	3.00	4.19	—
<sup>20</sup> Ne	-155.5	44.0	86.8	3.00	4.38	—
<sup>37</sup> Ar	-312.3	-43.1	-74.6	3.41	1.89	-1.159
<sup>36</sup> Ar	-301.9	-43.1	-84.8	3.41	2.50	—

Table II: Values of quadrupole deformation parameters in p-space and in r-space from Nilsson model calculations with ( $\Delta N \neq 0$ ), and without ( $\Delta N = 0$ ) major shell admixtures, compared to results of DDHF calculations.

	<sup>21</sup> Ne		<sup>20</sup> Ne		<sup>37</sup> Ar		<sup>36</sup> Ar	
	$\beta^p$	$\beta^r$	$\beta^p$	$\beta^r$	$\beta^p$	$\beta^r$	$\beta^p$	$\beta^r$
DDHF	0.041	0.402	0.050	0.405	0.014	-0.145	-0.011	-0.170
Nilsson <sup><math>\Delta N \neq 0</math></sup>	0.022	0.445	-0.002	0.487	0.028	-0.138	0.012	-0.171
Nilsson <sup><math>\Delta N = 0</math></sup>	0.232	0.232	0.234	0.234	-0.062	-0.062	-0.084	-0.084